Altitude

Visible Horizon A small circle on the surface of the earth, containing the points of intersection between the observer's eye sight and the surface of the earth.

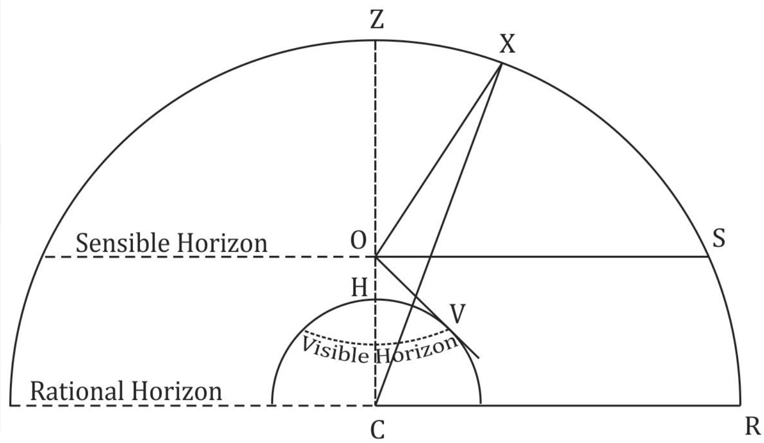
Rational Horizon A great circle on the celestial sphere, on a plane that passes through the centre of the earth and makes a right angle to the vertical of the observer.

Sensible Horizon A small circle on the celestial sphere, on a plane that passes through the observer's eye and is parallel to the rational horizon.

True Altitude The arc of the vertical circle on the celestial sphere contained between the heavenly body and the rational horizon.

Dip The angle at the observer's eye between the plane of the sensible horizon and the line joining the observer's eye and the visible horizon.

Polar Distance The angular distance of the celestial body from the pole.



O	Observer's eyes	SOV	Dip
X	Celestial body	XOV	Observer's altitude
C	Centre of the earth	XOS	Apparent altitude
OH	Height of eyes	XCR	True altitude

Altitude Corrections
Index Errors (IE) When the altitude of a heavenly body is taken by the sextant, the error of the sextant, if any, must be applied in order to obtain the true reading of the sextant, which is *observed altitude* (XOV). If the index error is on the arc, subtract it; if off the arc, then add it.

– on the arc
+ off the arc

Dip Corrections

Find the altitude of the body above the sensible horizon, which is *apparent altitude* (XOS). The difference between observed altitude and apparent altitude is *Dip* (SOV).

$$\text{XOS} = \text{XOV} - \text{SOV}$$
$$\text{Apparent Altitude} = \text{Observer Altitude} - \text{Dip}$$

So *Dip is always negative*. The amount of correction depends on the height of the observer's eye and is calculated by the formula:

$$\text{Dip} = -1.758'\sqrt{\text{height of eye in metres}}$$
$$\text{or}$$
$$\text{Dip} = -0.971'\sqrt{\text{height of eye in feet}}$$

Dip correction can be extracted from Norie's Nautical Tables or the Nautical Almanac.

Refraction

The light of the heavenly body is refracted; therefore, the altitude must be corrected according to the *refraction*. The refraction always causes the altitude to be greater than it is, so refraction correction is *always subtracted*. The amount of refraction depends on the altitude; it is maximum when the altitude is zero, and zero when the altitude is 90°. The refraction correction given in Norie's nautical table, which is Mean Refraction, is based on a temperature of 10°C and pressure of 1000mb, so additional correction might be necessary if the temperature and pressure differ from these values.

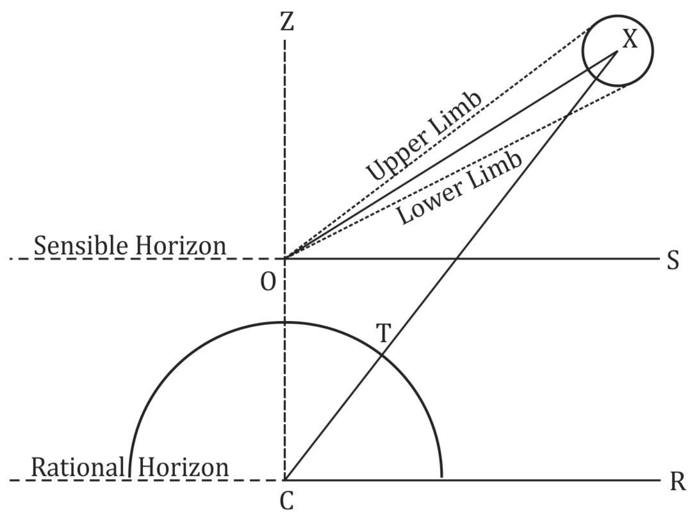
The altitude correction tables given in the Nautical Almanac are based on standard atmospheric conditions where the temperature is 10°C and pressure is 1010mb. If the atmosphere is not under standard conditions, then additional correction is required; this additional condition is also provided in the Nautical Almanac.

The refraction correction can also be simply calculated by formula:

$$\text{Refraction} = \frac{0.96}{\tan(\text{Apparent Altitude})} \qquad (\text{For altitude} > 11°)$$

Semi-Diameter

Since the sun and moon are large, it is difficult to measure the altitude from their centre, unlike stars or planets which appear as small dots. So, the altitude of the sun or moon is taken from either the lower or upper limb, then corrected for the body's radius, or *semi-diameter*, to obtain the altitude of the centre. The value of semi-diameter varies with earth's distance from the body. For the sun, its maximum is when the earth is nearest to the sun (December-January), and its minimum is when the earth is farthest away (June-July). For the moon, the value of the semi-diameter changes more rapidly due to the short duration of its revolution around the earth. If the altitude is measured from the upper limb, then the correction is subtracted; otherwise, it is added. The semi-diameter can be extracted from the daily page of the Nautical Almanac.

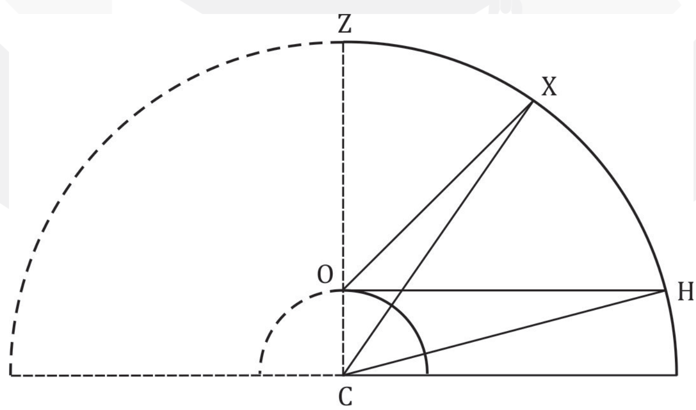


Semi-diameter can also be calculated from horizontal parallax by using following formula:

$$SD = \sin^{-1}\left(k \times \sin HP\right) \approx k \times HP \qquad k = 0.2725$$

k is the factor of the ratio of the radius of the moon (1738 km) to the equatorial radius of the earth (6378 km).

Parallax Parallax is the difference between the altitude of the heavenly body above the sensible horizon and the altitude above the rational horizon.



$$\text{Parallax} = XCR - XOS \qquad \because XCR = XTS \qquad \therefore \text{Parallax} = XTS - XOS$$

$$\text{From triangle } XOT\text{:} \quad XTS = XOT + OXT \quad \text{or} \quad XTS = XOS + OXC$$

$$\therefore \text{Parallax} = \left(XOS + OXC\right) - XOS$$

$$\therefore \text{Parallax} = OXC$$

Thus, the parallax is equal to the angle subtended by the earth's radius at the body and is **always positive**. Suppose the body is at position H, where the altitude is 0°; the parallax at H is OHC. Then the body rises to position X; the parallax now is OXC.

$$COH \text{ is a right-angled triangle:} \quad COH = 90° \quad \therefore OC = CH \sin OHC \quad (1)$$

$$\text{In triangle } COX\text{:} \quad \frac{\sin OXC}{OC} = \frac{\sin COX}{CX}$$

$$\text{Subtitute (1) for } OC\text{:} \quad \frac{\sin OXC}{CH \sin OHC} = \frac{\sin COX}{CX}$$

$$\because CH = CX \qquad \therefore \frac{\sin OXC}{\sin OHC} = \sin COX \quad \therefore \sin OXC = \sin OHC \sin COX$$

$$\because \sin COX = \sin ZOX \quad \text{(supplementary angles)}$$

$$\text{and } \sin ZOX = \cos XOH \quad \text{(complementary angles)}$$

$$\therefore \sin COX = \cos XOH$$

$$\sin OXC = \sin OHC \sin \cos XOH$$

sin(Parallax) = sin(Horizontal Parallax)cos(Altitude)

Thus, the value of correction of parallax varies with the altitude of the body, since cos0° equals 1 and cos90° equals 0, so it's maximum when the body is at the horizon and minimum when the altitude is 90°.

$$\sin(\text{Horizontal Paralax}) = \frac{OC}{CH} = \frac{\text{Earth's radius}}{\text{Distance of the body from the Earth}}$$

The distance of heavenly bodies from the earth is great in comparison with the earth's radius, so that the value of sin (Horizontal Parallax) is very small, and thus the correction for parallax is very small. This correction is negligible for stars, because they are too far away. For the sun, it's about 0.1´. But for the moon, because it's comparatively close to the earth, the value of horizontal parallax is greater compared with the sun or stars, so it must be calculated and applied to the apparent altitude to obtain more accurate true altitude. The parallax correction is obtained from Norie's Nautical Tables. For the moon, the horizontal parallax is obtained in the Nautical Almanac, in each hourly UT. This given horizontal parallax is **equatorial horizontal parallax** where it's the maximum. The horizontal parallax at a certain latitude can be found by using the reduction from the *"Reduction to the Moon's Horizontal Parallax for Latitude"* table.

Summary of altitude corrections for the sun

Sextant Altitude		
Index Error	– on the arc + off the arc	Sextant error
Observer Altitude		
Dip	–	Norie's Nautical Tables or Nautical Almanac
Apparent Altitude		
Refraction	–	Norie's Nautical Tables
Corr. for Temperature	if necessary	Norie's Nautical Tables
Corr. for Pressure	if necessary	Norie's Nautical Tables
Semi-Diameter	+ lower limb – upper limb	Daily page of Nautical Almanac
Parallax	+	Norie's Nautical Tables
True Altitude		

Example 1

The sextant altitudes were taken October 26, 2008, index error 1.2´ on the arc; height of eye 15.0 metres; temperature 29°; pressure 980 mb; and Sun's lower limb altitude 40°25.0´. Find the true altitude of the sun:

Sextant Altitude	40°25.0′	
Index Error	–1.2′	(on the arc)
Observed Altitude	40°23.8′	
Dip	–6.8′	(h.e. 15m)
Apparent Altitude	40°17.0′	
Refraction	–1.1'	
Corr. for Temperature	+0.1′	(temp. 29°C)
Corr. for Pressure	+0.0′	(pressure 980mb)
Semi-Diameter	+16.1′	
Paralax	+0.1′	
True Altitude	40°32.2′	

Example 2 On 19th July 2008, an officer observes the sun's upper limb to have a sextant altitude of 28°20′; index error is off the arc 2′; height of eye is 14m. Find the true altitude of the sun:

In this case, the temperature and pressure are not given, so normal atmosphere is assumed for refraction.

Sextant Altitude	28°20.0′	
Index Error	+2.0′	(off the arc)
Observed Altitude	28°22.0′	
Dip	−6.6′	(h.e. 14m)
Apparent Altitude	28°15.4′	
Refraction	−1.8′	
Semi-Diameter	−15.8′	(Upper limb)
Paralax	+0.1′	
True Altitude	27°57.9′	

Summary of altitude corrections for the moon

Sextant Altitude		
Index Error	– on the arc + off the arc	Sextant error
Observer Altitude		
Dip	–	Norie's Nautical Tables or Nautical Almanac
Apparent Altitude		
Refraction	–	Norie's Nautical Tables
Corr. for Temperature	if necessary	Norie's Nautical Tables
Corr. for Pressure	if necessary	Norie's Nautical Tables
Semi-Diameter	+ lower limb – upper limb	After apply augmentation against altitude to SD
Parallax	+	*Extract HP value from daily page of Nautical Almanac, apply reduction for latitude given in Norie's Nautical Tables, then apply formula* $\text{Parallax} = \text{HP} \times \cos(\text{Altitude})$
True Altitude		

Semi-Diameter calculation

$$\text{Augmented SD} = \text{SD} + \text{Augmentation}$$

Parallax calculation

$$\text{Reduced HP} = \text{HP} - \text{Reduction}$$

$$\text{Paralax} = \text{Reduced H.P.} \times \cos(\text{Apparent Altitude})$$

Example 3 Apparent altitude of the moon at latitude 40°N is 35°, the moon's semi-diameter and horizontal parallax are extracted from Nautical Almanac 15.8´ and HP 58´ respectively. Find semi-diameter and parallax corrections for altitude:

From Norie's Tables:

$$\left.\begin{aligned}\text{Latitude} &= 40^\circ\\ \text{SD} &= 15.8'\end{aligned}\right\}\text{Augmentation} = 0.2'$$

$$\text{Augmented SD} = \text{SD} + \text{Augmentation} = 15.8' + 0.2' = 16.0'$$

$$\left.\begin{aligned}\text{Latitude} &= 40^\circ\\ \text{HP} &= 58'\end{aligned}\right\}\text{Reduction} = 0.1'$$

$$\text{Reduced HP} = \text{HP} - \text{Reduction} = 58' - 0.1' = 57.9'$$

$$\begin{aligned}\text{Paralax} &= \text{Reduced H.P.} \times \cos(\text{Apparent Altitude})\\ &= 57.9' \times \cos 35^\circ = 47.4'\end{aligned}$$

Example 4 The sextant altitude 30°09.5´ of the moon's upper limb was taken at 1100 UT on October 26, 2008 in latitude 50°N, index error 1.8´ on the arc; height of eye 18 metres; temperature 15°C, pressure 960mb. Find the true altitude of the moon:

Extract from Nautical Almanac, at 1100 UT on October 26, 2008:

- Moon's SD: 15.3´
- Moon's HP: 56.1´

Sextant Altitude	30°09.5′	
Index Error	−1.8′	(on the arc)
Observer Altitude	30°07.7′	
Dip	−7.5′	(18m)
Apparent Altitude	30°00.2′	
Refraction	−1.7′	
Corr. for Temp.	+0.0′	(15°C)
Corr. for Pressure	+0.1′	(960mb)
SD	−15.4′	(Augmentation=0.1′)
Paralax	+48.6′	(HP=56.1′, Reduction= 0.1′)
True Altitude	30°31.8′	

Example 5 The sextant altitudes of the moon's lower limb were taken at 1700 UT on 18th July, 2008 in latitude 42°N; index error 1.3´ off the arc; height of eye 20 metres; temperature 25°C; pressure 1020mb. Sextant altitude is 15°28´. Find the true altitude of the moon:

Extract from Nautical Almanac, at 1700 UT on 18th July, 2008:

- Moon's SD: 15.0´
- Moon's HP: 55.2´

Sextant Altitude	15°28.0′	
Index Error	+1.3′	(off the arc)
Observer Altitude	15°29.3′	
Dip	–7.9′	(h.e. 20m)
Apparent Altitude	15°21.4′	
Refraction	–3.4'	
Corr. for Temp.	+0.3′	(25°C)
Corr. for Pressure	–0.1′	(1020mb)
SD	+15.2′	(Augmentation=0.2′)
Paralax	+53.1′	(HP=55.2′, Reduction= 0.1′)
True Altitude	16°26.5′	

Summary of altitude corrections for the stars and planets

Sextant Altitude		
Index Error	– on the arc + off the arc	Sextant error
Observer Altitude		
Dip	–	Nautical Almanac or Norie's Nautical Tables
Apparent Altitude		
Refraction	–	Norie's Nautical Tables
True Altitude		

Example 6

The sextant altitudes 45˚27.4´ of Star Bellatrix were taken October 26, 2008; index error 1.2´ on the arc; height of eye 15.0 metres. Find the true altitude of star Bellatrix:

Sextant Altitude	45°27.4′	
Index Error	–1.2′	(on the arc)
Observed Altitude	45°26.2′	
Dip	–6.8′	(h.e. 15m)
Apparent Altitude	45°19.4′	
Refraction	–0.9′	
True Altitude	45°18.5′	

Altitude Correction Table in Nautical Almanac

The Altitude Correction Tables in the Nautical Almanac give the combined correction for refraction, semi-diameter and parallax under standard atmosphere conditions, where atmospheric pressure is 1010mbs (29.5 ins) and temperature is 10˚C (50˚F). So, additional correction is required for refraction if atmospheric conditions are different.

For the sun

The correction is a combination of refraction, semi-diameter and parallax.

For the moon

The tables are divided two parts. The first part is a tabulated correction, which is the combination of refraction, semi-diameter and parallax for the lower limb, so if the altitude of the moon is taken from the upper limb, then 30´ must be subtracted. The second part is the correction for variations in semi-diameter and parallax, depending on the horizontal parallax.

For stars and planets

Basically, the correction is the refraction correction, and depending on the date, additional corrections might be required for Venus and Mars for parallax and phase.

Summary of altitude correction table for the sun in Nautical Almanac

Sextant Altitude		
Index Error	– on the arc + off the arc	
Observer Altitude		
Dip	–	
Apparent Altitude		
Main Correction	+ lower limb – upper limb	
Additional Correction	+ or –	For non-standard atmospheric condition
True Altitude		

Example 7

The sextant altitudes were taken October 26, 2008; index error 1.2´ on the arc; height of eye 15.0 metres; temperature 29°; pressure 980 mb; and Sun's lower limb altitude 40˚25.0´. Find the true altitude of the sun:

Sextant Altitude	40°25.0′	
Index Error	–1.2′	(on the arc)
Observed Altitude	40°23.8′	
Dip	–6.8′	(h.e. 15m)
Apparent Altitude	40°17.0′	
Correction	+15.1'	
Additional Correction	+0.1′	(temp. 29°C, pressure 980mb)
True Altitude	40°32.2′	

Summary of altitude correction table for the moon in Nautical Almanac

Sextant Altitude		
Index Error	– on the arc + off the arc	
Observer Altitude		
Dip	–	
Apparent Altitude		
Main Correction	+ Part 1	
	+ Part 2	Argument HP
Additional Correction	– 30´	for upper limb
	+ or –	For non-standard atmospheric condition
True Altitude		

Example 8

The sextant altitudes 30˚09.5´ of the moon's upper limb were taken at 1100 UT on October 26, 2008 in latitude 50°N; index error 1.8´ on the arc; height of eye 18 metres; temperature 15°C; pressure 960mb. Find the true altitude of the moon:

Sextant Altitude	30°09.5′	
Index Error	–1.8′	(on the arc)
Observed Altitude	30°07.7′	
Dip	–7.5'	(h.e. 18m)
Apparent Altitude	30°00.2′	
Main Correction (Part 1)	+58.9′	
(Part 2)	+2.7′	(upper limb, HP 56.1′)
Additional Correction	–30.0′	(upper limb)
	+0.1′	(temp. 15°C; pressure 960 mb)
True Altitude	30°31.9′	

Example 9

The sextant altitudes of the moon's lower limb were taken at 1700 UT on 18th July, 2008 in latitude 42°N; index error 1.3´ off the arc; height of eye 20 metres; temperature 25°C; pressure 1020mb. Sextant altitude is 15°28´. Find the true altitude of the moon:

From the Nautical Almanac, at 1700 UT on 18th July, 2008:
Moon's HP is 55.2´

Sextant Altitude	15°28.0′	
Index Error	+1.3′	(off the arc)
Observed Altitude	15°29.3′	
Dip	–7.9′	(h.e. 20m)
Apparent Altitude	15°21.4′	
Main Correction (Part 1)	+62.8′	
(Part 2)	+1.9′	(lower limb, HP 55.2′)
Additional Correction	+0.2′	(temp. 25°C, pressure 1020 mb)
True Altitude	16°26.3′	

Summary of altitude correction table for the star and planets in Nautical Almanac

Sextant Altitude		
Index Error	– on the arc + off the arc	Sextant error
Observer Altitude		
Dip	–	
Apparent Altitude		
Main Correction	–	
Additional Correction		For Venus and Mars
	+ or –	For non-standard atmospheric condition
True Altitude		

Example 10

The sextant altitude 45˚27.4´ of Star Bellatrix was taken October 26, 2008; index error 1.2´ on the arc; height of eye 15.0 metres; standard atmospheric conditions. Find the true altitude of Bellatrix:

Sextant Altitude	45°27.4′	
Index Error	–1.2′	(on the arc)
Observed Altitude	45°26.2′	
Dip	–6.8′	(h.e. 15m)
Apparent Altitude	45°19.4′	
Main Correction	–1.0′	
Additional Correction	0.0′	
True Altitude	45°18.4′	

Example 11 On 15th April 2008, the sextant altitude of Venus 37°46.8´ was taken; index error 1.5´ on the arc; height of eye 15.0 metres; standard atmospheric conditions. Find the true altitude of Venus:

Sextant Altitude	37°46.8′	
Index Error	–1.5′	(on the arc)
Observed Altitude	37°45.3′	
Dip	–6.8′	(h.e. 15m)
Apparent Altitude	37°38.5′	
Main Correction	–1.3′	
Additional Correction	+0.1′	
True Altitude	37°37.3′	

Norie's Altitude Total Correction Tables

The Total Correction Tables in the Norie's Nautical Tables are the combined correction for dip, refraction, semi-diameter and parallax. It's just simple interpolation between *observed altitude* and *height of eye*. All these corrections are based on standard atmosphere, and some insignificant corrections are omitted. They are convenient for the navigator to use, but for a high degree of accuracy, other correctional factors are necessary.

Sun's Total Correction

The correction is based on the assumed semi-diameter of 16.0´; additional corrections might be necessary, depending on the month of the year.

Sextant Altitude		
Index Error	– on the arc + off the arc	
Observer Altitude		
Total Correction	+ lower limb – upper limb	
Additional Correction	+ or –	For S.D.
True Altitude		

Example 12 The sextant altitudes were taken October 26, 2008; index error 1.2´ on the arc; height of eye 15.0 metres; temperature 29°; pressure 980 mb; Sun's lower limb altitude 40°25.0´. Find the true altitude of the sun:

Sextant Altitude	40°25.0′	
Index Error	−1.2′	(on the arc)
Observed Altitude	40°23.8′	
Total Correction	+8.3′	(h.e. 15m)
Additional Correction	+0.0′	(month: October)
True Altitude	40°32.1′	

Moon's Total Correction The correction is based on height of eye 49 metres (161 feet), and equatorial horizontal parallax. No reduction in latitude of horizontal parallax is applied, since it's considered insignificant.

Sextant Altitude		
Index Error	– on the arc + off the arc	
Observer Altitude		
Main Correction	+ lower limb + upper limb	
Additional Correction	– 30´	For upper limb
	+	Height of eye
True Altitude		

Example 13 The sextant altitudes of the moon's lower limb were taken at 1700 UT on 18th July, 2008 in latitude 42°N; index error 1.3´ off the arc; height of eye 20 metres; temperature 25°C; pressure 1020mb. Sextant altitude is 15°28´. Find the true altitude of the moon:

From the Nautical Almanac, at 1700 UT on 18th July, 2008: HP is 55.2´

Sextant Altitude	15°28.0′	
Index Error	+1.3′	(off the arc)
Observed Altitude	15°29.3′	
Main Correction	+52.6′	(h.e. 15m)
Additional Correction	+4.4′	(h.e. 20m)
True Altitude	16°26.3′	

Star's Total Correction

The correction in the *Star's Total Correction* Table is a combined correction for dip and refraction. It can also be used for planets, but for Venus and Mars, additional corrections given in the Nautical Almanac might be necessary, depending on the date.

Sextant Altitude		
Index Error	– on the arc + off the arc	Sextant error
Observer Altitude		
Dip	–	
Apparent Altitude		
Total Correction	–	
Additional Correction		For Venus and Mars
	+ or –	For non-standard atmospheric condition
True Altitude		

Example 14

The sextant altitudes 65°03.0´ of Star Diphda were taken on 15th April 2008, index error 2.5´ on the arc; height of eye 18.0 metres; standard atmospheric conditions. Find the true altitude of Diphda:

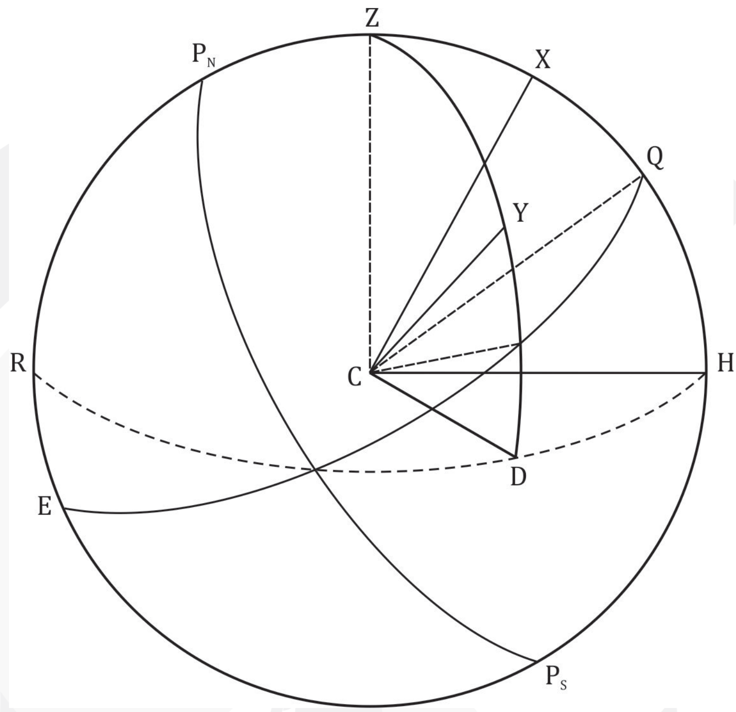
Sextant Altitude	65°03.0′	
Index Error	–2.5′	(on the arc)
Observed Altitude	65°00.5′	
Total Correction	–8.0′	
Additional Correction	+0.0′	
True Altitude	64°52.5′	

Example 15

Star Hadar was observed on 17th July 2008 with sextant altitudes of 18°31.0´; index error 3.0´ on the arc; height of eye 9.0 metres; standard atmospheric conditions. Find the true altitude of Hadar:

Sextant Altitude	18°31.0′	
Index Error	–3.0′	(on the arc)
Observed Altitude	18°28.0′	
Total Correction	–8.1′	
Additional Correction	+0.0′	
True Altitude	18°19.9′	

Zenith Distance Zenith distance of a body is the angular distance from the body to the observer's zenith, where zenith is the point on the celestial sphere projected directly above the observer.



Z	Zenith distance	EQ	The equinoctial
RH	Rational horizon	P_N & P_S	Poles
XH	Altitude of X	ZX	Zenith distance
YD	Altitude of Y	ZY	Zenith distance of Y
ZQ	Latitude of observer	XQ	Declination of X

If the celestial body is at position Y, then YD is the altitude of the body and ZY is the zenith distance of the body. Therefore:

Zenith Distance = Complement of Altitude

If the celestial body is at position X, in the same meridian as the observer, then XH is the altitude of the body, ZX is the zenith distance, and XQ is the declination of the body, but ZQ is the latitude of the observer, so when the body is on the same meridian as the observer, there is a relationship between the latitude of the observer, the declination of the heavenly body and the zenith distance. Therefore, when the heavenly body is bearing 000˚ or 180˚ (due North or South), by knowing the declination of the heavenly body (which can be extracted from the Nautical Almanac) and the zenith distance (which is the complement of altitude of the body measured by sextant), the latitude of the observer can be calculated.