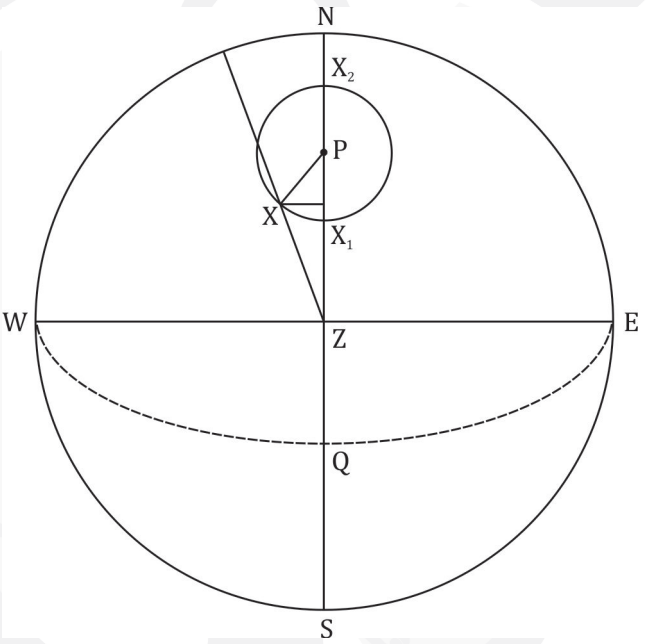
Latitude by Polaris (Pole Star)

Polaris is a star close to the North Pole; it circles around the pole at a distance or radius of 1°. In the figure below:

$$\left.\begin{aligned} NZ &= PQ = 90° \\ NZ &= NP + PZ \\ PQ &= QZ + PZ \end{aligned}\right\} \therefore NP = QZ$$

$\Rightarrow$ Altitude of North Pole = Latitude of Observer



If Polaris coincides with the north celestial pole, then the altitude of Polaris is the latitude of the observer. However, Polaris is not coinciding with the pole but is circling around the pole. In the diagram, X's circle represents the daily path of Polaris around the pole.

$$\text{At } X_1: \quad \text{Altitude of Polaris: } NX_1 \quad \Rightarrow \text{Latitude} = NX_1 - PX_1$$

$$\text{At } X_2: \quad \text{Altitude of Polaris: } NX_2 \quad \Rightarrow \text{Latitude} = NX_2 + PX_2$$

At position X_1, the correction is negative with the quantity as the distance from Polaris to the pole; as it goes further in an anticlockwise direction, the correction quantity gets smaller but is still negative up to a one-instant position where the correction become nil; then, the quantity of the correction becomes positive and increases until it is on transit with the pole and with the observer (X_2), where the correction is positive and is equal to the distance from Polaris to the pole. Continuing to move to the west side of the observer's meridian, the result is similar, except that the correction is from positive

to negative until the star is back to position X_1. At any instant for position X, the correction is PY, which very much depends on the local hour angle (ZPX) of Polaris.

In the Nautical Almanac, the correction is tabulated in three quantities, a_0, a_1 and a_2. They contain a constant that will give a positive value in all cases; the sum of the constants is 1°. Hence:

$$\text{Latitude} = \text{True Altitude} + a_0 + a_1 + a_2 - 1°$$

Procedure for obtaining the latitude by Polaris altitude

1. Obtain UT of Aries at the time of observation;
2. Find LHA of Aries by using following formula:

$$\text{LHA}^{\Upsilon} = \text{GHA}^{\Upsilon} \begin{matrix} +\text{East Longitude} \\ -\text{West Longitude} \end{matrix}$$

3. Find true altitude of Polaris from sextant altitude;
4. Use Pole Star Table to find correction a_0, a_1, a_2 and azimuth of Polaris by using the LHA^{Υ} and DR latitude. If DR latitude is unknown, then apparent altitude or true latitude can be used;
5. Add corrections a_0, a_1 and a_2 to true altitude of Polaris and subtract 1° to obtain the latitude of the observer.

Note

1. Correction a_0 is needed to interpolate if necessary, but corrections a_1 and a_2 are unnecessary for interpolation.
2. The azimuth is used to find the position line which lies at right angles to the azimuth.
3. The position line is running 90° to the bearing or azimuth of star; generally, the bearing should not exceed 2 ½ ° off the bearing of the North Pole, therefore the position line may be taken to run 90°/270°.

Example 1 On 15th April 2008, at DR position 46°30′N 46°15′W, at $08^h\ 58^m$ UT, Polaris was bearing 002° with sextant altitude 46°30.4′; index error 2.0′ on the arc; height of eye 15 m. Find latitude of observer and compass error:

GHA♈ at $15^d\ 08^h00^m$	323°52.1′	Sextant Altitude	46°30.4′
Increments (58^m)	14°32.4′	Index Error	−2.0′
GHA♈ at $15^d\ 08^h58^m$	338°23.6′	Observed Altitude	46°28.4′
Longitude (W)	46°15.0′	Dip	−6.9′
LHA♈	292°08.6′	Apparent Altitude	46°21.5′
		Total Correction	−0.9′
		True Altitude	46°20.6′
		a^0	+1°12.2′
		a^1	+0.6′
Azimuth	001.0°	a^2	+0.2′
Compass Bearing	002.0°		−1°
Compass Error	1.0°W	Latitude:	46°33.6′ N

Example 2 On 25th October 2008, in DR position 32°22.9′N. 31°20′E., at $03^h\ 30^m\ 05^s$ UT, Polaris was bearing 358.1°, the sextant altitude was 32°42.4′; index error 1.6′ on the arc, height of eye 14 m. Find latitude, compass error and position line:

GHA♈ $03^h00^m00^s$	78°52.7′	Sextant Altitude	32°42.4′
Increments (30^m05^s)	7°32.5′	Index Error	−1.6′
GHA♈ $03^h30^m05^s$	86°25.2′	Observed Altitude	32°40.8′
Longitude (E)	31°20.0′	Dip	−6.6′
LHA♈	117°45.2′	Apparent Altitude	32°34.2′
		Total Correction	−1.5′
		True Altitude	32°32.7′
		a^0	+0°49.9′
		a^1	+0.5′
Azimuth	359.2°	a^2	+0.3′
Compass Bearing	358.1°		−1°
Compass Error	1.1°E	Latitude:	32°23.4′ N

The position line passes through position 32°23.4′N 31°20.0′E perpendicular with azimuth, therefore the position line is 269.2° / 89.2°.