Longitude by Chronometer

This method is used to determine the longitude and the position line which runs through the position of DR latitude and observed longitude.

Procedure to obtain longitude and position line by chronometer

1. Determine the UT when taking the sight;
2. Obtain GHA and declination from Nautical Almanac;
3. Find true altitude of celestial body;
4. Determine P, which is the d. long. between celestial body and observer by using the formula:

$$P = \cos^{-1}\left(\frac{\sin(\text{True Altitude}) - \sin \text{Lat}.\sin \text{Dec}.}{\cos \text{Lat}.\cos \text{Dec}.}\right)$$

   If the name of the latitude and declination is contrary, then the declination is treated as a negative quantity.

5. Sketch diagram to indicate the relation between DR longitude and GHA to determine LHA. Inspect whether the sight is taken before or after the meridian passage. If before, the celestial body is east of the observer; if after, the celestial body is west of observer.
   - Before meridian passage: $\text{LHA} = 360 - \text{P}$
   - After meridian passage: $\text{LHA} = \text{P}$
6. Determine observed longitude either by inspection diagram, or by the formula:

$$\text{Longitude West} = \text{GHA} - \text{LHA}$$
$$\text{Longitude East} = \text{LHA} - \text{GHA}$$

7. Using ABC tables or formula to determine the azimuth of celestial body from observer:

$$A = \frac{\tan \text{Lat}.}{\tan \text{P}} \qquad B = \frac{\tan \text{Dec}.}{\sin \text{P}} \qquad C = A \pm B$$

$$\text{Azimuth} = \tan^{-1}\left(\frac{1}{C \times \cos \text{Lat}}\right)$$

8. Determine the position line which is perpendicular to the azimuth; the position line through which the position line passes is DR latitude and calculated longitude.

Example 1

Longitude by chronometer - Sun

In the morning on 24th October 2008, in DR position 23°15′N 148°42.0′W, the sextant altitude of the Sun's upper limb was 30°10.0′, index error 2.1′ on the arc, height of eye 15 metres. When the chronometer showed $05^h32^m10^s$, the chronometer error was 02^m01^s fast. Find the position line and the position through which it runs:

Chronometer	$05^h32^m10^s$	Longitude	148°42′W
Error (fast)	2^m01^s	Zone	+10
Chronometer	$05^h30^m09^s$	∴ UT 24^d	$17^h30^m09^s$

GHA	78°58.4′	Sextant Altitude	30°10.0′
Increment	7°32.3′	Index Error	−2.1′
GHA	86°30.7′	Observed Altitude	30°07.9′
		Dip	−6.9′
		Apparent Altitude	30°01.0′
Declination	12°03.0′N	Correction	−17.7′
d = 0.9	0.5′	True Altitude	29°43.3′
	12°03.5′N	TZD	60°16.7′

$$P = \cos^{-1}\left(\frac{\sin(\text{True Altitude}) - \sin \text{Lat}.\sin \text{Dec.}}{\cos \text{Lat}.\cos \text{Dec.}}\right)$$

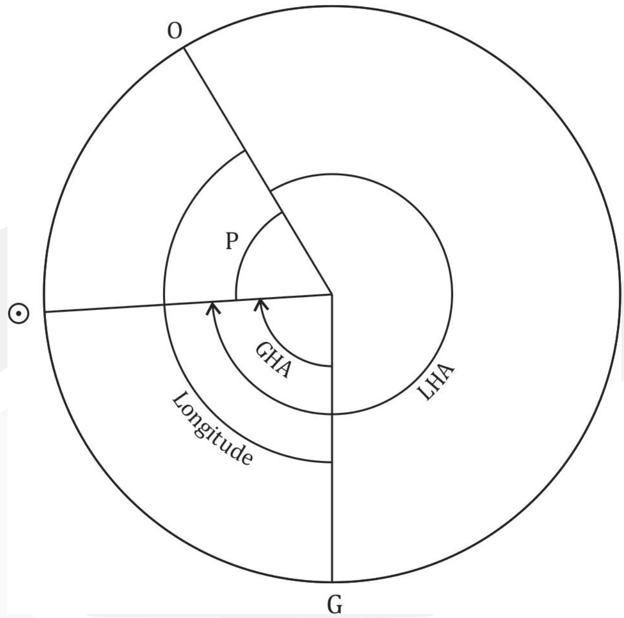
$$= \cos^{-1}\left(\frac{\sin 29°43.3' - \sin 23°15' \sin 12°03.5'}{\cos 23°15' \cos 12°03.5'}\right)$$

$$= 62°36.8'$$

$$\left.\begin{aligned} \text{DR longitude} &= 148°42'\text{W} \\ \text{GHA} &= 86°30.7' \end{aligned}\right] \text{Before mer pass}$$

$$\therefore \quad \text{LHA} = 360° - P = 360° - 62°36.8' = 297°23.2'$$

$$\begin{aligned} \text{Longitude (W)} &= \text{GHA} - \text{LHA} \\ &= 86°30.7' - 297°23.2' \\ &= (86°30.7' + 360) - 297°23.2' \\ &= 149°07.5'\text{W} \end{aligned}$$



$$A = \frac{\tan Lat.}{\tan P} = \frac{\tan 23°15'}{\tan 62°36.8'} = 0.222574S$$

$$B = \frac{\tan Dec.}{\sin P} = \frac{\tan 12°03.5'}{\sin 62°36.8'} = 0.240585N$$

A	0.222574	S
B	0.240585	N
C	0.018011	N

$$AZ = \tan^{-1}\left(\frac{1}{C \times \cos Lat}\right) = \tan^{-1}\left(\frac{1}{0.018011 \times \cos 23°15'}\right)$$
$$= N89.1°E$$
$$= 089.1°T$$

Position line runs 179.1° / 359.1°
through position 23°15´N 149°07.5´W

Example 2

Longitude by chronometer - Moon

On 17th July 2008, at 1930, in DR position 20°15´S 114°24´W, the sextant altitude of the Moon's upper limb was 28°27.5´, index error 1.2´ off the arc, height of eyes 18 metres. Chronometer was $03^h13^m20^s$, with error 03^m05^s fast. Find the position line and the position through which it passes:

Approx. LMT	17^d	$19^h30^m00^s$	Chronometer	$03^h13^m20^s$
Longitude (W)		$07^h37^m36^s$	Error (fast)	3^m05^s
Approx. UT	18^d	$03^h07^m36^s$	UT (18^{th})	$03^h10^m15^s$

GHA	45°30.2′	Sextant Altitude	28°27.5′
Increment	2°26.7′	Index Error (off)	+1.2′
v = 11.2	2.0′	Observed Altitude	28°28.7′
GHA	47°58.9′	Dip	−7.5′
		Apparent Altitude	28°21.2′
Declination	23°34.8′S	Main Correction (Part 1)	+59.6′
d = 7.8	1.4′	(Part 2)	+1.9′
	23°33.4′S	Additional Correction	−30′
		True Altitude	28°52.7′

$$P = \cos^{-1}\left(\frac{\sin(\text{True Altitude}) - \sin \text{Lat.} \sin \text{Dec.}}{\cos \text{Lat.} \cos \text{Dec.}}\right)$$

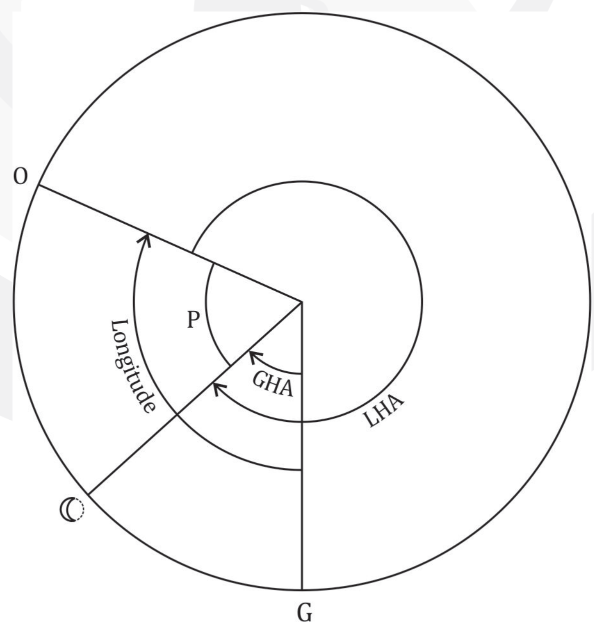
$$= \cos^{-1}\left(\frac{\sin 28°52.7' - \sin 20°15' \sin 23°33.4'}{\cos 20°15' \cos 23°33.4'}\right)$$

$$= 66°22.6'$$

$$\left.\begin{aligned} \text{DR longitude} &= 114°24'\text{W} \\ \text{GHA} &= 47°58.9' \end{aligned}\right] \text{Before mer pass}$$

$$\therefore \quad \text{LHA} = 360° - P = 360° - 66°22.6' = 293°37.4'$$

$$\begin{aligned} \text{Longitude (W)} &= \text{GHA} - \text{LHA} \\ &= 47°58.9' - 293°37.4' \\ &= (47°58.9' + 360) - 293°37.4' \\ &= 114°21.5' \end{aligned}$$



$$A = \frac{\tan Lat.}{\tan P} = \frac{\tan 20°15'}{\tan 66°22.6'} = 0.161356N$$

$$B = \frac{\tan Dec.}{\sin P} = \frac{\tan 23°33.4'}{\sin 66°22.6'} = 0.475867S$$

A	0.161356	N
B	0.475867	S
C	0.314511	S

$$AZ = \tan^{-1}\left(\frac{1}{C \times \cos Lat}\right) = \tan^{-1}\left(\frac{1}{0.314511 \times \cos 20°15'}\right)$$

$$= S73.6°E$$

$$= 106.4°T$$

Position line runs 016.4° / 196.4°
through position 20°15´N 114°21.5´W

Example 3

Longitude by chronometer - Star

On 15th April 2008, evening, in DR position 30°42´N 60°30´W, the sextant altitude of the star Regulus was 45°32.5´, index error 2.2´ on the arc, height of eyes 15 metres. The chronometer showed $06^h28^m00^s$, with error 02^m10^s slow. Find the direction of position line and the position through which it passes:

Chronometer	$09^h28^m00^s$	Longitude		60°30′W
Error (slow)	2^m10^s	Zone		+4
Chronometer	$09^h30^m10^s$	∴ UT	15^d	$21^h30^m10^s$

GHA^♈	159°23.2′	Sextant Altitude	45°32.5′
Increment	7°33.7′	Index Error	−2.2′
GHA^♈	166°56.9′	Observed Altitude	45°30.3′
SHA*	207°47.5′	Dip	−6.8′
GHA*	14°44.4′	Apparent Altitude	45°23.5′
		Correction	−1.0′
Declination	11°55.5′ N	True Altitude	45°22.5′

$$P = \cos^{-1}\left(\frac{\sin(\text{True Altitude}) - \sin Lat.\sin Dec.}{\cos Lat.\cos Dec.}\right)$$

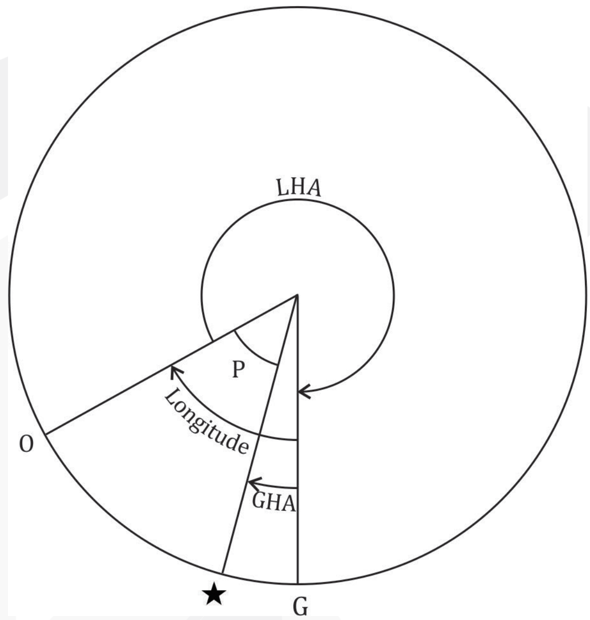
$$= \cos^{-1}\left(\frac{\sin 45°22.5' - \sin 30°42' \sin 11°55.5'}{\cos 30°42' \cos 11°55.5'}\right)$$

$$= 45°52.4'$$

$$\left.\begin{array}{r} \text{DR longitude} = 60°30'W \\ GHA = 14°44.4' \end{array}\right] \text{Before mer pass}$$

$$\therefore \quad LHA = 360° - P = 360° - 45°52.4' = 314°07.6'$$

$$
\begin{aligned}
\text{Longitude (W)} &= \text{GHA} - \text{LHA} \\
&= 14°44.4' - 314°07.6' \\
&= (14°44.4' + 360) - 314°07.6' \\
&= 60°36.8'
\end{aligned}
$$



$$
A = \frac{\tan \text{Lat.}}{\tan P} = \frac{\tan 30°42'}{\tan 45°52.4'} = 0.575926\text{S}
$$

$$
B = \frac{\tan \text{Dec.}}{\sin P} = \frac{\tan 11°55.5'}{\sin 45°52.4'} = 0.294216\text{N}
$$

A	0.575926	S
B	0.294216	N
C	0.281710	S

$$
AZ = \tan^{-1}\left(\frac{1}{C \times \cos \text{Lat}}\right) = \tan^{-1}\left(\frac{1}{0.281710 \times \cos 30°42'}\right)
$$

$$
= \text{S}76.4°\text{E}
$$

$$
= 103.6°\text{T}
$$

Position line runs 013.6° / 193.6°
through position 30°42´N 60°36.8´W

Example 4

Longitude by chronometer - Planet

On 22nd July 2008, in DR position 11°50´S 070°00´E, the sextant altitude of Mars was 40°28.5´, index error 1.5´ on the arc, height of eyes 18 metres. The chronometer showed $01^h20^m56^s$, with error 2^m40^s fast. Find the direction of the position line and the position through which it passes:

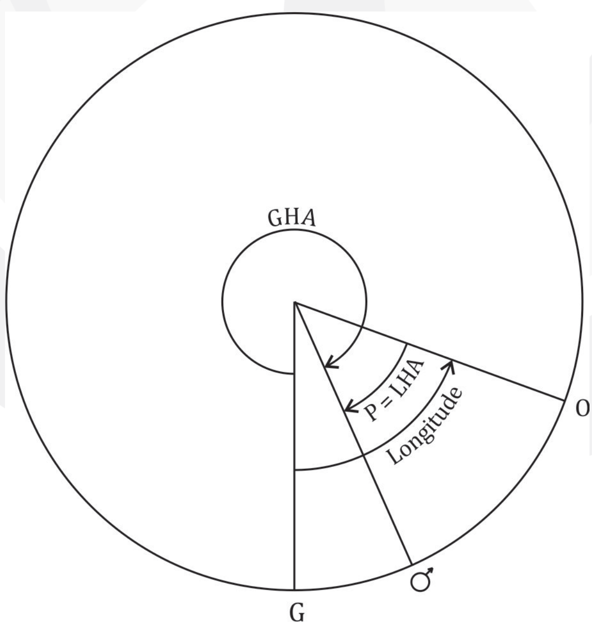
Chronometer	$01^h20^m56^s$	Longitude	70°00′E
Error (slow)	2^m40^s	Zone	–5
Chronometer	$01^h18^m16^s$	∴ UT 22ᵈ	$13^h18^m16^s$

GHA	331°20.7′	Sextant Altitude	40°28.5′
Increment	4°34.0′	Index Error (on)	+1.5′
v = 1.0	0.3′	Observed Altitude	40°27.0′
GHA	335°55.0′	Dip	–7.5′
		Apparent Altitude	40°19.5′
Declination	7°40.6′N	Main Correction	–1.1′
d = 0.6	0.2′	Additional Correction	+0.1′
	7°40.4′N	True Altitude	40°18.5′

$$P = \cos^{-1}\left(\frac{\sin(\text{True Altitude}) - \sin \text{Lat.} \sin \text{Dec.}}{\cos \text{Lat.} \cos \text{Dec.}}\right)$$

$$= \cos^{-1}\left(\frac{\sin 40°18.5' - \sin 11°50' \sin(-7°40.4')}{\cos 11°50' \cos(7°40.4')}\right)$$

$$= 45°57.7'$$



$$\left.\begin{aligned} \text{DR longitude} &= 70°00'\text{E} \\ \text{GHA} &= 335°55.0' \end{aligned}\right] \text{ After mer pass}$$

$$\therefore \quad \text{LHA} = \text{P} = 45°57.7'$$

$$\begin{aligned} \text{Longitude (E)} &= \text{LHA} - \text{GHA} \\ &= 45°57.7' - 335°55.0' \\ &= (45°57.7' + 360) - 335°55.0' \\ &= 70°02.7' \end{aligned}$$

$$\left.\begin{aligned} A &= \frac{\tan \text{Lat.}}{\tan \text{P}} = \frac{\tan 11°50'}{\tan 45°57.7'} = 0.202600\text{N} \\ B &= \frac{\tan \text{Dec.}}{\sin \text{P}} = \frac{\tan 7°40.4'}{\sin 45°57.7'} = 0.187420\text{N} \end{aligned}\right\} \begin{array}{ll} \text{A} & 0.202600\,\text{N} \\ \text{B} & \underline{0.187420}\,\text{N} \\ \text{C} & 0.390020\,\text{N} \end{array}$$

$$\begin{aligned} \text{AZ} &= \tan^{-1}\left(\frac{1}{\text{C} \times \cos \text{Lat}}\right) = \tan^{-1}\left(\frac{1}{0.390020 \times \cos 11°50'}\right) \\ &= \text{N}69.1°\text{W} \\ &= 290.9°\text{T} \end{aligned}$$

Position line runs 020.9° / 200.9°
through position 11°50´S 70°02.7´E