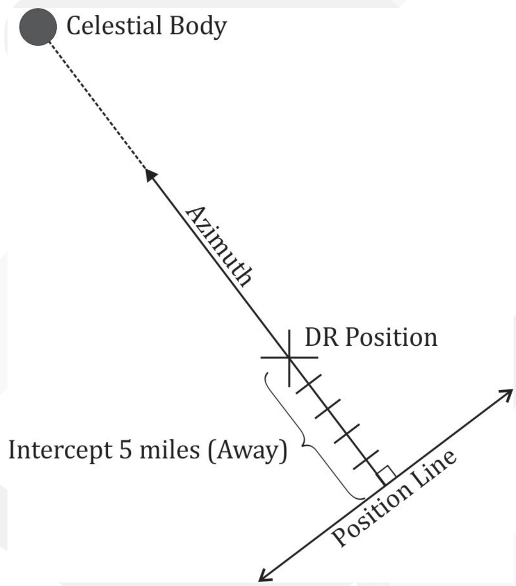
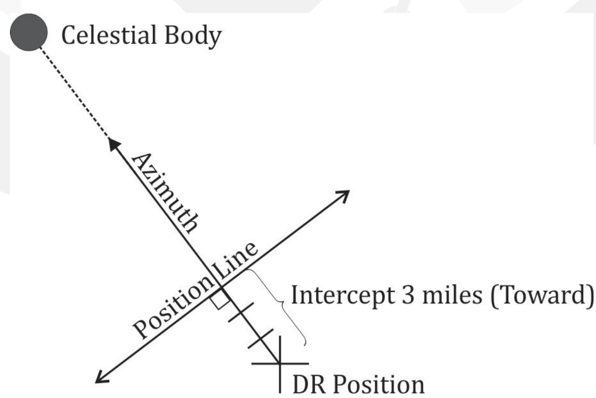
The Marqc St. Hilaire Method

Sometimes called the **Intercept Method,** this method solves the PZX triangle to find zenith distance and azimuth, which are the angular distance between zenith and celestial body, and the bearing of a celestial body. This zenith distance is called **Calculated Zenith Distance**; it will be compared with true zenith distance, which is obtained from corrected true altitude. The difference between the calculated and true zenith distances is called **intercept**, and is indicated either **toward** or **away** from a celestial body. If the true zenith distance is less than the calculated zenith distance, the intercept is called **toward;** if the true zenith distance is greater than calculated zenith distance, the intercept is called **away.**

$$\text{True Zenith Distance} \langle \text{ Calculated Zenith Distance} \Rightarrow \text{Toward}$$
$$\text{True Zenith Distance} \rangle \text{ Calculated Zenith Distance} \Rightarrow \text{Away}$$





Example 1 The sun was observed bearing 150°T at altitude 51°20´. The calculated zenith distance is 38°42´ for the DR position 44°12´N 125° 20´E. Find the position through which to plot the position line (Intercept Terminal Point):

True Altitude	51°20′
	90°
True Zenith Distance	38°40′
Calculated Zenith Distance	38°42′
Intercept	2′ (Toward)

From Traverse Tables:

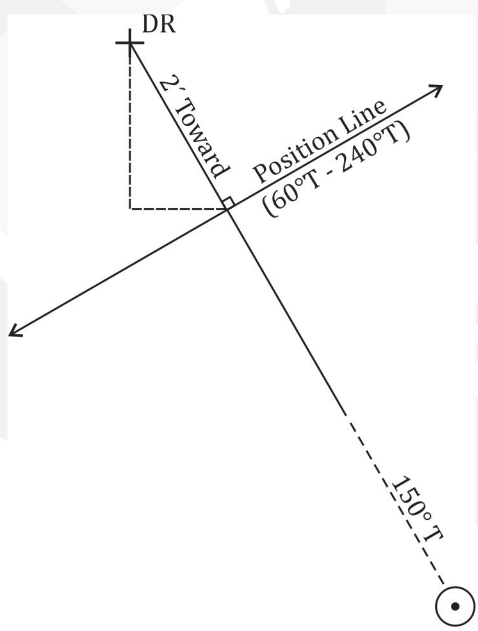
$$\left.\begin{array}{r}\text{Bearing } 150^\circ \text{ or } S30^\circ E\\ \text{Distance } 2'\end{array}\right] \Rightarrow \begin{array}{l}\text{D.Lat.} = 1.7'\,(S)\\ \text{Dep.} = 1'(E)\end{array}$$

$$\text{Mean Lat. } (\text{Lat.}_m) = 44^\circ 12' N - (1.7' \div 2) = 44^\circ 11.2'$$

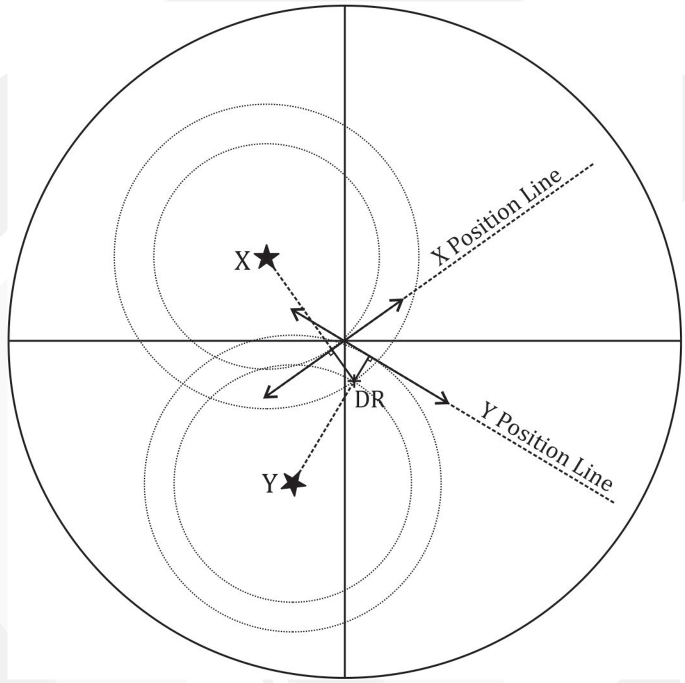
$$\text{D. Long.} = \text{Dep.} \times \cos \text{Lat.}_m = 1' \times \cos 44^\circ 11.2' = 1.4'(E)$$

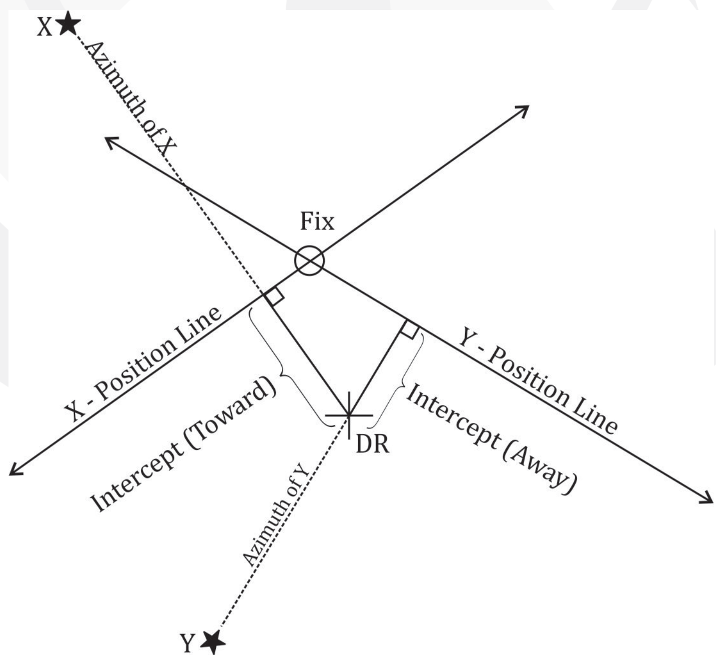
D.R. Lat.	44°12.0′N	D.R. Long.	125°20.0′E
D. Lat.	1.7′ (S)	D. Long.	1.4′ (E)
Lat.	44°10.3′ N	Long.	125°21.4′ E

ITP = 44°10.3′N 125°21.4′E



The figures below showing celestial bodies X and Y are sighted from the observer's position. The body X has a true zenith distance less than calculated zenith distance; the intercept is **toward**, while body Y has a true zenith distance greater than calculated zenith distance, and the intercept is **away**. The intersection of the two position lines is the fix or the zenith of the observer.





Procedures to obtain position by the Marqc St. Hilaire Method

1. Find UT;
2. From the Nautical Almanac, extract GHA and declination of celestial body;
3. Apply DR longitude to obtain LHA of the body;
4. Draw figure diagram;
5. Calculate zenith distance by using following formula:

$$\cos CZD = \cos PZ \cos PX + \sin PZ \sin PX \cos LHA$$
$$\therefore \quad CZD = \cos^{-1}\left(\cos PZ \cos PX + \sin PZ \sin PX \cos LHA\right)$$

or

$$\cos CZD = \sin Lat.\sin Dec. + \cos Lat.\cos Dec.\cos LHA$$
$$\mathbf{CZD = \cos^{-1}\left(\sin Lat.\sin Dec. + \cos Lat.\cos Dec.\cos LHA\right)}$$

For above formula, if the name of latitude of observer is contrary to the declination of the celestial body, then the declination of celestial body is treated as a negative quantity.

6. From sextant altitude, find true altitude and obtain True Zenith Distance (TZD);
7. Compare CZD and TZD to obtain the intercept and name it as follows:

$$\text{True Zenith Distance} \langle \text{ Calculated Zenith Distance} \Rightarrow \text{Toward}$$
$$\text{True Zenith Distance} \rangle \text{ Calculated Zenith Distance} \Rightarrow \text{Away}$$

8. Find azimuth of the body by using ABC tables, or the following formula:

$$\mathbf{\tan AZ = \frac{\sin LHA}{\tan Dec.\cos Lat. - \cos LHA \sin Lat.}}$$

$$\mathbf{AZ = \tan^{-1}\left(\frac{\sin LHA}{\tan Dec.\cos Lat. - \cos LHA \sin Lat.}\right)}$$

For above formula, if the name of latitude of observer is contrary to the declination of the celestial body, then the declination of celestial body is treated as a negative quantity.

- If denominator is negative, azimuth will be named South (S).

- If denominator is positive, azimuth will be named North (N).
- If LHA is between 0° and 180°, azimuth will be named West (W).
- If LHA is between 180° and 360°, azimuth will be named East (E).

Example 2 In the evening, 17th July 2008, at DR position 40°25´N, 32°40´W, the chronometer showed $10^h19^m17^s$, chronometer error 4^m09^s fast. Observed Star Dubhe with sextant altitude 43°32.0´, and Star Deneb with sextant altitude 38°12.3´; index error 2.3´ on the arc; height of eye 15 m. Find intercepts and position lines:

Chronometer	$10^h19^m17^s$	Longitude	32°40′W
Error	4^m09^s	Zone	+2
Chronometer	$10^h15^m08^s$	UT 17^d	$22^h15^m08^s$

Star Dubhe

GHA^{Υ}	266°05.6′	Declination	61°42.5′ N
Increment	3°47.6′		
GHA^{Υ}	269°53.2′		
SHA	193°56.3′		
GHA^{*}	103°49.5′		
Longitude (W)	32°40.0		
LHA	71°09.5′		

$$\text{CZD} = \cos^{-1}\left(\sin\text{Lat}.\sin\text{Dec}. + \cos\text{Lat}.\cos\text{Dec}.\cos\text{LHA}\right)$$

$$= \cos^{-1}\left(\sin 40°25'\sin 61°42.5' + \cos 40°25'\cos 61°42.5'\cos 71°09.5'\right)$$

$$= 46°34.4'$$

Sextant Altitude	43°32.0′	TZD	46°38.1′
Index Error	−2.3′	CZD	46°34.4′
Observed Altitude	43°29.7′	Intercept	3.7′ Away
Dip	−6.8′		(TZD>CZD)
Apparent Altitude	43°22.9′		
Correction	−1.0′		
True Altitude	43°21.9′		

$$AZ = \tan^{-1}\left(\frac{\sin LHA}{\tan Dec.\cos Lat. - \cos LHA \sin Lat.}\right)$$

$$= \tan^{-1}\left(\frac{\sin 71°09.5'}{\tan 61°42.5' \cos 40°25' - \cos 71°09.5' \sin 40°25'}\right)$$

$$= N38.1°W$$

$$= 321.9°T$$

Position line runs 051.9°T / 231.9°T

Star Deneb

GHA♈	269°53.2′	Declination	45°18.6′ N
SHA	49°33.6′		
GHA*	319°26.8′		
Longitude (W)	32°40.0′		
LHA	286°46.8′		

$$CZD = \cos^{-1}\left(\sin Lat.\sin Dec. + \cos Lat.\cos Dec.\cos LHA\right)$$

$$= \cos^{-1}\left(\sin 40°25' \sin 45°18.6' + \cos 40°25' \cos 45°18.6' \cos 286°46.8'\right)$$

$$= 52°00.7'$$

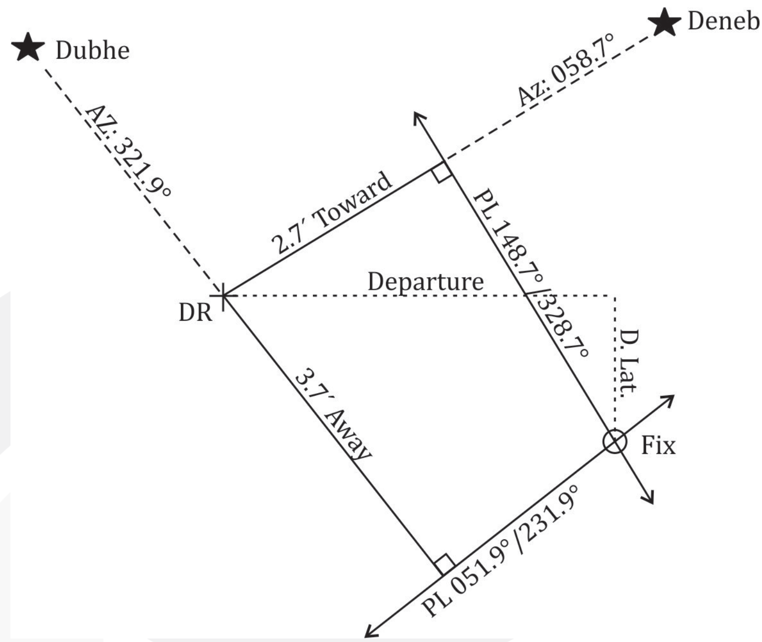
Sextant Altitude	38°12.3′	TZD	51°58.0′
Index Error	–2.3′	CZD	52°00.7′
Observed Altitude	38°10.0′	Intercept	2.7′ Toward
Dip	–6.8′		$(TZD < CZD)$
Apparent Altitude	38°03.2′		
Correction	–1.2′		
True Altitude	38°02.0′		

$$AZ = \tan^{-1}\left(\frac{\sin LHA}{\tan Dec.\cos Lat. - \cos LHA \sin Lat.}\right)$$

$$= \tan^{-1}\left(\frac{\sin 286°46.8'}{\tan 45°18.6' \cos 40°25' - \cos 286°46.8' \sin 40°25'}\right)$$

$$= N58.7°E = 058.7°T$$

Position line runs 148.7°T / 328.7°T



Example 3 At 0900, 25th October 2008, DR position 43°15′N, 38°25′W, the chronometer shows $11^h40^m32^s$, chronometer error is 2^m20^s slow. Sextant altitude of the sun's lower limb is 24°02.3′; index error 1.5′ off the arc; height of eye 12 m. Find intercept and position line:

Approx. LMT	9^h00^m	Chronometer	$11^h40^m32^s$
Long. (W)	$2^h33^m40^s$	Error (slow)	2^m20^s
Approx. UT	$11^h33^m40^s$	Chronometer	$11^h42^m52^s$
		UT	$11^h42^m52^s$

GHA	348°59.7′	Declination	12°18.5′ S
Increment	10°43.0′	d = 0.9′	0.6′
GHA	359°42.7′		12°19.1′ S
Longitude (W)	38°25.0′		
LHA	321°17.7′		

$$
\begin{aligned}
CZD &= \cos^{-1}\left(\sin Lat.\sin Dec. + \cos Lat.\cos Dec.\cos LHA\right) \\
&= \cos^{-1}\begin{pmatrix}\sin 43°15'\sin(-12°19.1') \\ +\cos 43°15'\cos(-12°19.1')\cos 321°17.7'\end{pmatrix} \\
&= 65°51.0'
\end{aligned}
$$

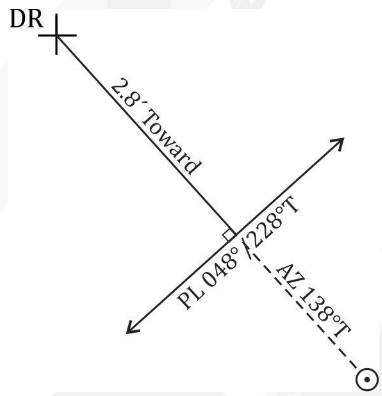
Sextant Altitude	24°02.3′	TZD	65°48.2′
Index Error	+1.5′	CZD	65°51.0′
Observed Altitude	24°03.8′	Intercept	2.8′ Toward (TZD <CZD)
Dip	−6.1′		
Apparent Altitude	23°57.7′		
Correction	+14.1′		
True Altitude	24°11.8′		

$$AZ = \tan^{-1}\left(\frac{\sin LHA}{\tan Dec.\cos Lat. - \cos LHA \sin Lat.}\right)$$

$$= \tan^{-1}\left(\frac{\sin 321°17.7'}{\tan(-12°19.1')\cos 43°15' - \cos 321°17.7' \sin 43°15'}\right)$$

$$= S42.0°E = 138°T$$

Position line runs 048°T / 228°T



Procedure to find fix position when using Intercept Method

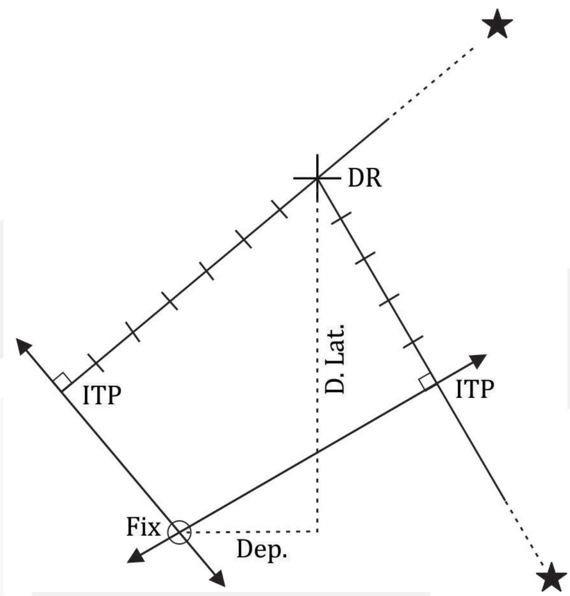
1. Plot DR position;

2. From DR position, draw the bearing line of the first body then mark intercept with chosen scale, either away from or toward the body from DR position. At intercept terminal point (ITP), draw the position line that is perpendicular to the bearing of the body;

3. Repeat step 2 for second body. The intersection of two position lines is the fix position of observer;

4. From DR position, measure D. Lat. and departure; then calculate mean latitude to convert departure into D. Long;

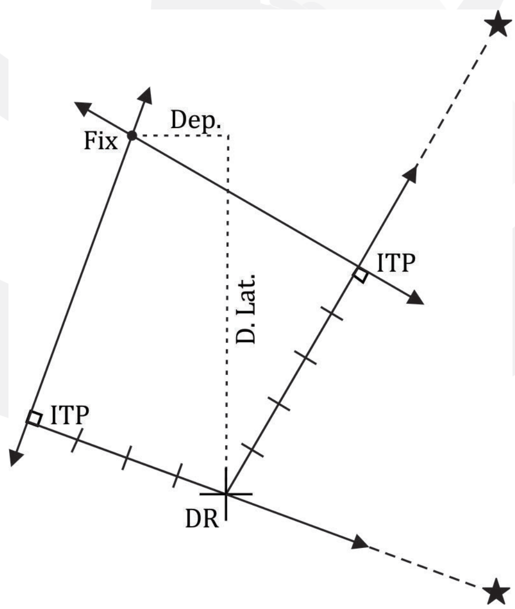
5. Apply D. Lat. and D. Long. to DR position to obtain fix position.



Example 4 At DR position, 19°20´N, 116°50´E., two observations of stars were taken as follows:

1. Bearing 110° T, intercept 4´ away.
2. Bearing 030° T, intercept 5´ toward.

Find the ship's position:



By measurement:

$$\begin{array}{rlrll} \text{D.Lat.} = & 6.8' & \text{DR Lat.} & 19°20.0'\,\text{N} & \\ \text{Dep.} = & 1.8' & \text{D.Lat.} & \underline{\quad 6.8'}\ (\text{N}) & \\ & & \text{Fix Lat.} & 19°26.8'\,\text{N} & \text{Lat.}_m \quad 19°23.4'\,\text{N} \end{array}$$

$$\text{D.Long.} = \frac{\text{Dep.}}{\cos \text{Lat.}_m} = \frac{1.8'}{\cos 19°23.4'} = 1.9'(\text{W})$$

$$\begin{array}{rlrl} \text{DR Long.} & 116°50.0'\,\text{E} & \text{Fix Position:} & \\ \text{D.Long.} & \underline{\quad 1.9'(\text{W})} & \text{Latitude} & 19°26.8'\text{N} \\ \text{Fix Long.} & 116°48.1'\,\text{E} & \text{Longitude} & 116°48.1'\,\text{E} \end{array}$$