The Marqc St. Hilaire Method

Sometimes called the Intercept Method, this method solves the PZX triangle to find zenith distance and azimuth, which are the angular distance between zenith and celestial body, and the bearing of a celestial body. This zenith distance is called **Calculated Zenith Distance**; it will be compared with true zenith distance, which is obtained from corrected true altitude. The difference between the calculated and true zenith distances is called intercept, and is indicated either toward or away from a celestial body. If the true zenith distance is less than the calculated zenith distance, the intercept is called toward; if the true zenith distance is greater than calculated zenith distance, the intercept is called away.

True Zenith Distance ‹ Calculated Zenith Distance ⇒ Toward
True Zenith Distance › Calculated Zenith Distance ⇒ Away
Example 1  The sun was observed bearing 150°T at altitude 51°20′. The calculated zenith distance is 38°42′ for the DR position 44°12′N 125° 20′E. Find the position through which to plot the position line (Intercept Terminal Point):

<table>
<thead>
<tr>
<th>True Altitude</th>
<th>51°20′</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Zenith Distance</td>
<td>38°40′</td>
</tr>
<tr>
<td>Calculated Zenith Distance</td>
<td>38°42′</td>
</tr>
<tr>
<td>Intercept</td>
<td>2′ (Toward)</td>
</tr>
</tbody>
</table>

From Traverse Tables:

Bearing 150° or S30°E  \[ \Rightarrow \]  D.Lat. = 1.7′ (S)
Distance 2′  \[ \Rightarrow \]  Dep. = 1′ (E)

Mean Lat. (Lat.\(_m\)) = 44°12′N − (1.7′ ÷ 2) = 44°11.2′
D. Long. = Dep. × cosLat.\(_m\) = 1′ × cos44°11.2′ = 1.4′ (E)

D.R. Lat. 44°12.0′N  \[ \Rightarrow \]  D.R. Long. 125°20.0′E
D. Lat. 1.7′ (S)  \[ \Rightarrow \]  D. Long. 1.4′ (E)
Lat. 44°10.3′N  \[ \Rightarrow \]  Long. 125°21.4′E

ITP = 44°10.3′N  125°21.4′E
The figures below showing celestial bodies X and Y are sighted from the observer's position. The body X has a true zenith distance less than calculated zenith distance; the intercept is toward, while body Y has a true zenith distance greater than calculated zenith distance, and the intercept is away. The intersection of the two position lines is the fix or the zenith of the observer.
Procedures to obtain position by the Marqc St. Hilaire Method

1. Find UT;
2. From the Nautical Almanac, extract GHA and declination of celestial body;
3. Apply DR longitude to obtain LHA of the body;
4. Draw figure diagram;
5. Calculate zenith distance by using following formula:
   \[ \cos CZD = \cos PZ \cos PX + \sin PZ \sin PX \cos LHA \]
   \[ \therefore CZD = \cos^{-1} \left( \cos PZ \cos PX + \sin PZ \sin PX \cos LHA \right) \]
   or
   \[ \cos CZD = \sin \text{Lat}. \sin \text{Dec.} + \cos \text{Lat}. \cos \text{Dec.} \cos LHA \]
   \[ CZD = \cos^{-1} \left( \sin \text{Lat}. \sin \text{Dec.} + \cos \text{Lat}. \cos \text{Dec.} \cos LHA \right) \]
   
   For above formula, if the name of latitude of observer is contrary to the declination of the celestial body, then the declination of celestial body is treated as a negative quantity.

6. From sextant altitude, find true altitude and obtain True Zenith Distance (TZD);
7. Compare CZD and TZD to obtain the intercept and name it as follows:
   True Zenith Distance (Calculated Zenith Distance) ⇒ Toward
   True Zenith Distance (Calculated Zenith Distance) ⇒ Away

8. Find azimuth of the body by using ABC tables, or the following formula:
   \[ \tan AZ = \frac{\sin \text{LHA}}{\tan \text{Dec.} \cos \text{Lat.} - \cos \text{LHA} \sin \text{Lat.}} \]
   \[ AZ = \tan^{-1} \left( \frac{\sin \text{LHA}}{\tan \text{Dec.} \cos \text{Lat.} - \cos \text{LHA} \sin \text{Lat.}} \right) \]
   
   For above formula, if the name of latitude of observer is contrary to the declination of the celestial body, then the declination of celestial body is treated as a negative quantity.

■ If denominator is negative, azimuth will be named South (S).
- If denominator is positive, azimuth will be named North (N).
- If LHA is between $0^\circ$ and $180^\circ$, azimuth will be named West (W).
- If LHA is between $180^\circ$ and $360^\circ$, azimuth will be named East (E).

Example 2
In the evening, 17th July 2008, at DR position $40^\circ25´N$, $32^\circ40´W$, the chronometer showed $10^h19^m17^s$, chronometer error $4^m09^s$ fast. Observed Star Dubhe with sextant altitude $43^\circ32.0´$, and Star Deneb with sextant altitude $38^\circ12.3´$; index error $2.3´$ on the arc; height of eye $15$ m. Find intercepts and position lines:

<table>
<thead>
<tr>
<th>Chronometer</th>
<th>Longitude</th>
<th>Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^h19^m17^s$</td>
<td>$32^\circ40´W$</td>
<td>+2</td>
</tr>
<tr>
<td>Error</td>
<td>$4^m09^s$</td>
<td></td>
</tr>
<tr>
<td>Chronometer</td>
<td>UT</td>
<td>UT 17d 22h15m08s</td>
</tr>
</tbody>
</table>

**Star Dubhe**

<table>
<thead>
<tr>
<th>GHA °′</th>
<th>Declination</th>
</tr>
</thead>
<tbody>
<tr>
<td>$266^\circ05.6´$</td>
<td>$61^\circ42.5´N$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Increment</th>
<th>SHA °′</th>
<th>GHA °′</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^\circ47.6´$</td>
<td>$193^\circ56.3´$</td>
<td>$103^\circ49.5´$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Longitude (W)</th>
<th>LHA °′</th>
</tr>
</thead>
<tbody>
<tr>
<td>$32^\circ40.0$</td>
<td>$71^\circ09.5´$</td>
</tr>
</tbody>
</table>

\[CZD = \cos^{-1}(\sin\text{Lat.}\sin\text{Dec.} + \cos\text{Lat.}\cos\text{Dec.}\cos\text{LHA})\]

\[= \cos^{-1}(\sin 40^\circ25´\sin 61^\circ42.5´ + \cos 40^\circ25´\cos 61^\circ42.5´\cos 71^\circ09.5´)\]

\[= 46^\circ34.4´\]

<table>
<thead>
<tr>
<th>Sextant Altitude</th>
<th>TZD °′</th>
</tr>
</thead>
<tbody>
<tr>
<td>$43^\circ32.0´$</td>
<td>$46^\circ38.1´$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Index Error</th>
<th>CZD °′</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2.3´$</td>
<td>$46^\circ34.4´$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Observed Altitude</th>
<th>Intercept</th>
<th>Away</th>
</tr>
</thead>
<tbody>
<tr>
<td>$43^\circ29.7´$</td>
<td>$3.7´$</td>
<td>(TZD&gt;CZD)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dip</th>
<th>Apparent Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-6.8´$</td>
<td>$43^\circ22.9´$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correction</th>
<th>True Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.0´$</td>
<td>$43^\circ21.9´$</td>
</tr>
</tbody>
</table>
AZ = \tan^{-1}\left( \frac{\sin LHA}{\tan Dec. \cdot \cos Lat. - \cos LHA \cdot \sin Lat.} \right)

= \tan^{-1}\left( \frac{\sin 71^\circ 09.5'}{\tan 61^\circ 42.5' \cdot \cos 40^\circ 25' - \cos 71^\circ 09.5' \cdot \sin 40^\circ 25'} \right)

= N 38.1^\circ W

= 321.9^\circ T

Position line runs 051.9^\circ T / 231.9^\circ T

\textit{Star Deneb}

\begin{align*}
\text{GHA}^\circ & = 269^\circ 53.2' \\
\text{SHA} & = 49^\circ 33.6' \\
\text{GHA}^\circ & = 319^\circ 26.8' \\
\text{Longitude (W)} & = 32^\circ 40.0' \\
\text{LHA} & = 286^\circ 46.8'
\end{align*}

\begin{align*}
\text{CZD} & = \cos^{-1}\left( \sin Lat. \cdot \sin Dec. + \cos Lat. \cdot \cos Dec. \cdot \cos LHA \right) \\
& = \cos^{-1}\left( \sin 40^\circ 25' \cdot \sin 45^\circ 18.6' + \cos 40^\circ 25' \cdot \cos 45^\circ 18.6' \cdot \cos 286^\circ 46.8' \right) \\
& = 52^\circ 00.7'
\end{align*}

\begin{align*}
\text{Sextant Altitude} & = 38^\circ 12.3' \\
\text{Index Error} & = -2.3' \\
\text{Observed Altitude} & = 38^\circ 10.0' \\
\text{Dip} & = -6.8' \\
\text{Apparent Altitude} & = 38^\circ 03.2' \\
\text{Correction} & = -1.2' \\
\text{True Altitude} & = 38^\circ 02.0'
\end{align*}

\begin{align*}
\text{AZ} & = \tan^{-1}\left( \frac{\sin LHA}{\tan Dec. \cdot \cos Lat. - \cos LHA \cdot \sin Lat.} \right) \\
& = \tan^{-1}\left( \frac{\sin 286^\circ 46.8'}{\tan 45^\circ 18.6' \cdot \cos 40^\circ 25' - \cos 286^\circ 46.8' \cdot \sin 40^\circ 25'} \right) \\
& = N 58.7^\circ E = 058.7^\circ T
\end{align*}

Position line runs 148.7^\circ T / 328.7^\circ T
Example 3

At 0900, 25\textsuperscript{th} October 2008, DR position 43\textdegree{}15´N, 38\textdegree{}25´W, the chronometer shows 11\textdegree{}40\textquoteright{}32\textquoteright{}, chronometer error is 2\textquoteright{}20\textquoteright{} slow. Sextant altitude of the sun's lower limb is 24\textdegree{}02.3´; index error 1.5´ off the arc; height of eye 12 m. Find intercept and position line:

\begin{align*}
\text{Approx. LMT} & \quad 9^h 00^m \\
\text{Long. (W)} & \quad 2^h 33^m 40^s \\
\text{Error (slow)} & \quad 2^m 20^s \\
\text{Approx. UT} & \quad 11^h 33^m 40^s \\
\text{Chronometer} & \quad 11^h 42^m 52^s \\
\text{UT} & \quad 11^h 42^m 52^s \\
\end{align*}

\begin{align*}
\text{GHA} & \quad 348.59.7' \\
\text{Declination} & \quad 12\textdegree{}18.5'S \\
\text{Increment} & \quad 10.43.0' \quad \text{d} = 0.9' \quad 0.6' \\
\text{GHA} & \quad 359.42.7' \quad 12\textdegree{}19.1'S \\
\text{Longitude (W)} & \quad 38.25.0' \\
\text{LHA} & \quad 321.17.7' \\
\end{align*}

\begin{align*}
\text{CZD} = & \cos^{-1}(\sin \text{Lat.} \sin \text{Dec.} + \cos \text{Lat.} \cos \text{Dec.} \cos \text{LHA}) \\
= & \cos^{-1}\left(\sin 43\textdegree{}15\textquoteright{}\sin(-12\textdegree{}19.1\textquoteright{}) + \cos 43\textdegree{}15\textquoteright{}\cos(-12\textdegree{}19.1\textquoteright{})\cos 321\textdegree{}17.7\textquoteright{}\right) \\
= & 65\textdegree{}51.0\textquoteright{}
\end{align*}
### Sextant Altitude

<table>
<thead>
<tr>
<th></th>
<th>24°02.3’</th>
<th>TZD 65°48.2’</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Index Error</strong></td>
<td>+1.5’</td>
<td>CZD 65°51.0’</td>
</tr>
<tr>
<td><strong>Observed Altitude</strong></td>
<td>24°03.8’</td>
<td>Intercept 2.8’ Toward</td>
</tr>
<tr>
<td><strong>Dip</strong></td>
<td>-6.1’</td>
<td>(TZD &lt; CZD)</td>
</tr>
<tr>
<td><strong>Apparent Altitude</strong></td>
<td>23°57.7’</td>
<td></td>
</tr>
<tr>
<td><strong>Correction</strong></td>
<td>+14.1’</td>
<td></td>
</tr>
<tr>
<td><strong>True Altitude</strong></td>
<td>24°11.8’</td>
<td></td>
</tr>
</tbody>
</table>

### Procedure to find fix position when using Intercept Method

1. Plot DR position;
2. From DR position, draw the bearing line of the first body then mark intercept with chosen scale, either away from or toward the body from DR position. At intercept terminal point (ITP), draw the position line that is perpendicular to the bearing of the body;
3. Repeat step 2 for second body. The intersection of two position lines is the fix position of observer;
4. From DR position, measure D. Lat. and departure; then calculate mean latitude to convert departure into D. Long;
5. Apply D. Lat. and D. Long. to DR position to obtain fix position.

\[
\begin{align*}
AZ &= \tan^{-1}\left(\frac{\sin LHA}{\tan \text{Dec.} \cos \text{Lat.} - \cos LHA \sin \text{Lat.}}\right) \\
&= \tan^{-1}\left(\frac{\sin 321°17.7’}{\tan(-12°19.1’)\cos 43°15’ - \cos 321°17.7’\sin 43°15’}\right) \\
&= S42.0° E = 138° T
\end{align*}
\]

Position line runs 048° T / 228° T
Example 4  At DR position, 19°20´N, 116°50´E., two observations of stars were taken as follows:

1. Bearing 110° T, intercept 4´ away.
2. Bearing 030° T, intercept 5´ toward.

Find the ship’s position:
By measurement:

\[
\begin{align*}
\text{D.Lat.} &= 6.8' \quad \text{DR Lat.} \quad 19^\circ 20.0' \ N \\
\text{Dep.} &= 1.8' \quad \text{D.Lat.} \quad 6.8' \ (N) \\
\text{Fix Lat.} &= 19^\circ 26.8' \ N \quad \text{Lat.}_m \quad 19^\circ 23.4' \ N
\end{align*}
\]

\[
\begin{align*}
\text{D.Long.} &= \frac{\text{Dep.}}{\cos \text{Lat.}_m} = \frac{1.8'}{\cos 19^\circ 23.4'} = 1.9' \ (W)
\end{align*}
\]

DR Long. \quad 116^\circ 50.0' \ E \quad \text{Fix Position:} \quad \begin{align*}
\text{D.Long.} \quad 1.9' \ (W) \quad \text{Latitude} \quad 19^\circ 26.8' \ N \\
\text{Fix Long.} \quad 116^\circ 48.1' \ E \quad \text{Longitude} \quad 116^\circ 48.1' \ E
\end{align*}